Combining taper and basic wood density equations for estimating stem biomass of the *Populus x canadensis* I – 488 variety

Combinación de funciones de ahusamiento y densidad básica de la madera para la estimación de la biomasa fustal de *Populus* x *canadensis* variedad I – 488

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SUMMARY

Stem biomass estimates are useful for various purposes. For example, the amount of commercially usable timber, carbon stock and energy fuel potential are directly related to biomass quantity. This paper describes the development of a biomass flexible equation for poplar (*Populus x canadensis* I - 488) combining stem profile and basic wood density models, which were fitted and validated with a sample of 60 trees of this variety of poplar. The biomass equation, which is obtained from the double integral of the product between the basic wood density and cross-sectional stem area, was analyzed in four scenarios given by combining average and individual tree models for both stem profile and basic wood density. The most accurate results in both stem profile and basic wood density modeling, as in the stem biomass estimates, were obtained with individual tree models.

Key words: flexible equation, individual tree equation, stem profile, simultaneous fitting, complementary mass-volume quantification.

RESUMEN

La cuantificación de la biomasa fustal es útil para variados objetivos. Entre otros, permite estimar la cantidad de madera comercialmente aprovechable, cantidad de carbono almacenado y cantidad potencial de combustible energético que se puede obtener de la biomasa. Este documento describe el desarrollo de una función flexible de biomasa para la variedad de álamo *Populus* x *canadensis*, variedad I-488, combinando modelos de perfil fustal y de densidad básica de la madera, los que se ajustaron y validaron con una muestra de 60 árboles. La función flexible de biomasa, que se obtiene de la doble integral del producto entre la densidad básica de la madera y del área de sección transversal del fuste, se evaluó en cuatro escenarios diferentes, resultantes de combinar modelos promedio y de árbol individual tanto de perfil fustal como de densidad de la madera. Los resultados más exactos tanto en las modelizaciones del perfil fustal y de la densidad básica de la madera, como en la estimación de la biomasa, se obtuvieron con los modelos de árbol individual.

Palabras clave: función flexible, función de árbol individual, perfil fustal, ajuste individual simultáneo, cuantificación complementaria másica-volumétrica.

INTRODUCTION

The stem of trees accumulates the major portion of the above ground lignocellulosic biomass. Thus, determination of stem biomass is of fundamental interest for various purposes. Among others, biomass can be used as an indicator of the amount of timber available for commercial use (*e.g.* lumber, chips or pulp), the amount of carbon stocks and potential amount of extractable energy fuel wood per unit area (Brown *et al*. 1989, Navar *et al*. 2001, Schlegel 2001). It can also be used to compare productivity among individuals, species and sites (Gaillard de Benitez *et al*. 2000, Segura and Kanninen 2005), in a determined area and time (Brown *et al*. 1989), allowing the design of suitable management schemes for plantations and natural forests.

The quantification of stem biomass can be performed by direct or indirect methods (Welles and Cohen 1996, Schlegel 2001, Fonseca *et al*. 2009). Direct methods entail destructive procedures involving the sectioning of the stem to obtain a sample of stem disks from which the dry mass of the stem is determined. One option is to use the ratio between the dry weight and the fresh weight of the biomass samples; the total dry weight of the stem or a fraction of this is obtained from the product between the weight ratio and the respective fresh weight of each tree. Another more elaborate option, which requires registering the height at which each stem disk was obtained and diameter of each stem disk, is to use the product of the volume and the basic density of the wood; the volume of each stem section, established between each pair of successive stem disks, is obtained from the product between the distance between the stem disks and the average of the respective cross-sectional areas; the basic wood density of the wood is obtained from the ratio of the dry weight of the stem

disk without bark and its fresh volume. The dry weight of each section is obtained from the product of the respective volume and the average basic density values; the total dry weight of the stem or a fraction thereof, which can be defined in terms of a merchantable top diameter, is obtained from the sum of the dry weights of the respective sections.

Indirect methods are a more practical alternative for estimating stem biomass. In general, they involve the use of allometric models which determine the relationship existing between the stem biomass and easily measurable variables, such as the diameter at breast height (DBH), i.e. the stem diameter measured at 1.30 m above ground level, and the total tree height (Brown *et al*. 1989, Myers 1990), or variables derived therefrom such as the square of the diameter and its product with total height (Dieguez *et al.* 2003). Diameter is one of the most suitable predictors for fitting of this type of models, owing to its high correlation with biomass (Specht and West 2003, Fonseca *et al*. 2009); in young individuals, it may be more appropriate to use the stump diameter instead (Husch 2001, Pérez and Kanninen 2002). Obviously, the fitting of the models is preceded by a destructive sampling which provides the necessary data (Brown *et al*. 1989, Diéguez *et al*. 2003, Muñoz *et al*. 2005). Although the equations resulting from the fitting permit to estimate the biomass with a high degree of accuracy, its greatest limitation derives from the fact that there are habitually produced equations for fractions of the stem between the stump and the upper part of the stem defined in terms of a fixed merchantable top diameter.

A flexible equation for estimating biomass can be produced from the independent modeling of basic wood density and stem profile. This equation, which allows estimating biomass between any two stem heights, is obtained from the double integral of the product between the stem sectional area, estimated by means of a stem profile model, and basic wood density (Parresol and Thomas 1989); variables modeled depending on the relative height of the stem. The model of Bruce (1968) in its basic form can be used in the modeling of the stem profile, *i.e.* to obtain an average profile model, and two variants of the same; the first consisting of estimating the exponents of the model from the data available for the poplar variety under study; the second, a single-tree model in which the coefficients of the equation are estimated from the size of the tree. Correspondingly, in the modeling of the basic wood density an average model and an individual tree model can be used. As a result, four scenarios are obtained resulting from combining the stem profile equations obtained from the average and the individual tree fitting with the two equations of basic wood density. We hypothesized that the more precise flexible biomass equation is obtained combining both stem taper and basic wood density equations based on individual-tree effects.

Although the fitting of both models requires the same information as used for fitting traditional biomass models with fixed merchantable top diameter, not only is the equation resulting from the double integral more flexible and useful than those for fixed merchantable top diameter, but also this, in conjunction with the stem profile model, constitutes a simple set of equations that allow the complementary volumetric-mass quantification of the stem. For example, the fraction of stem with dimensions suitable to generate logs destined for sawmills and rotated and laminated veneer may require a volumetric quantification; the remaining stem could be used for bioenergy and require quantification of the mass. Obviously, which fraction of the tree to allocate to what purpose is a matter of supply and demand, in which the price per unit volume or unit biomass plays a determining role. Complementary mass-volumetric quantification tools are essential to carry out this analysis.

The objective of this study is to generate a flexible equation for estimating the stem biomass of *Populus x canadensis* – variety I488 (poplar), which might be very useful for planning the integrated use of the stem, especially in those cases where the stem generates some products that require a volumetric quantification (*e.g.* logs destined for sawmills, rotated and laminated veneer), while others require a mass quantification (*e.g.* pulp, bioenergy).

To simplify the double-integral process we used the same explanatory variable for both stem taper and basic wood density equations. This relative stem height explanatory variable is adequate to reduce residual variability and to facilitate the fitting of average stem taper and average basic wood density models. Diameter at breast height (DBH), *i.e.* the stem diameter measured at 1.30 m above ground level, and the total tree height were used to evaluate individual-tree effects.

METHODS

Description of the database. The database used in this study was compiled in 2011 in four stands of the *Populus x canadensis* I - 488 variety, belonging to Compañía Agrícola y Forestal El Álamo (CAF El Álamo), located in The Retiro city, Maule region, Chile (table 1). They are stands established with ample distances of 6 m x 6 m, *i.e.* 278 trees per hectare, and maintained with that constant low density until harvest, which is carried out when stands are around 14 years old. In each stand 15 trees were selected, covering the entire diametric range recorded in an inventory conducted in 2010; twelve of these trees were used for the fitting and three, randomly selected, came to constitute the sample for validation of stem profile, basic wood density and stem biomass models.

Measured variables. In each one of the standing trees selected to constitute the sample the diameter at breast height (DBH) was recorded. Once felled, the stump height and total height were recorded. Then sections were cut into 3.3 m lengths, according to the quality and length of logs predefined by the CAF protocol for finished products, *i.e.*

Table 1. Descriptive statistics of the database used in models fitting and validation.
Estadísticos descriptivos de la base de datos usada en el ajuste y validación de los modelos.

Stand	Age (year)	Site Index[a] (m)	DBH (cm)				Total height (m)				Hc[b] (m)			
			Min.	Mean	Max.	VC%[c]	Min.	Mean	Max.	VC%	Min.	Mean	Max.	VC%
Sample used in models fitting														
1	16	24.73	23.5	32.7	38.0	12.8	29.3	33.2	36.2	7.0	12.6	17.2	22.7	15.2
2	15	24.28	23.1	29.5	34.1	8.8	28.6	30.5	32.1	3.0	14.2	16.5	18.0	7.7
3	14	34.35	30.9	34.8	40.0	6.9	33.8	35.3	36.6	2.3	15.3	18.9	20.8	8.5
4	14	29.70	28.0	32.6	36.0	9.0	33.4	34.9	36.1	2.3	12.1	16.7	20.9	15.0
Sample used in models validation														
1	16	24.73	26.8	29.6	31.7	7.1	30.4	31.6	33.2	3.8	15.9	16.7	17.5	4.0
2	15	24.28	21.6	26.6	30.6	13.8	29.0	29.6	30.2	1.7	13.4	15.7	17.1	11.1
3	14	34.35	34.6	36.8	40.8	7.7	35.0	35.2	35.5	0.7	15.9	18.5	20.1	10.3
4	14	29.70	30.2	32.4	34.7	5.8	33.8	35.0	35.7	2.4	17.6	19.4	20.3	6.4

[a] Mean height at base age 10 yrs.; [b] Height to living crown; [c] variation coefficient.

from the stump height and up to a top merchantable diameter not inferior to 15.2 cm; thence and up to a top merchantable diameter not less than 10.2 cm, 2.5 m logs were obtained. At the ends of each log, stem disks were obtained which were marked, labeled and stored for later processing in order to obtain their bark thickness, the basic density of the wood and the normal diameter over-bark, *i.e.* the diameter perpendicular to the ground in the position in which the stem of felled trees remained. In the middle of the length of each one of the logs, and at the heights of 80, 130, 180, 280 cm of the base log of each tree, the normal diameter over-bark was recorded. The bark thickness corresponding to the diameters measured in the logs was estimated by linear interpolation, using the measuring height and bark thickness from the stem disks obtained at the end of each log.

Basic wood density was determined from the ratio of the dry weight and the green volume of an angular sample in each one of the stem disks. Dry weight without bark was obtained with an electronic balance with a precision of 0.01 g, after dried in an oven at 105 °C, until reaching a constant weight. The green volume was determined by the liquid displacement method, with electronic balance with accuracy of 0.1 g, assuming water density of 1 g cm^{-3}. The angular sample comprised a sector angle of 30° with the center in the pith and comprising up to the outermost part of each stem disk and located on the average radius of the stem disk, which was obtained from a series of radial measurements.

Stem profile modeling. In the modeling of the stem profile three alternatives were considered. The first consisted of the average model fitting (model 1), which corresponds to

a basic model proposed by Bruce *et al.* (1968), in which fixed values were assigned to the exponents. The second consisted of fitting an average model, but estimating the exponents e_i of the model from the stem taper data available for this study. The third consisted of fitting an individual tree stem profile model, in which the coefficients b_i of the basic model were estimated based on the diameter at breast height (D), total height (H), height of live tree crown (H_c) and combinations thereof, *i.e.* $b_i = f$ (1 D H D^2 H^2 DH D^2H HC D^2H_c). This individual model took the best set of exponents as a base, from between the first two options.

Relative values were used to adjust the taper model into the model to express the incorporated variables. These variables can also be directly incorporated. However, it is more useful to define them as relative values, because the variability of the dependent variable decreases; a basic requirement of the traditional regression technique (Raymond *et al.* 1989), and it is easier to visualize the relationship among variables. In this way, the definition of the dependent variable was guided by the ultimate use of the taper function. This is because when transforming the dependent variable of the function set and the value of a specific variable needs to be estimated, bias can be introduced between any two points of the stem in the estimates.

$$y_i^{\,2} = b_1 x_i^{\,e_1} + b_2 x_i^{\,e_2} + b_3 x_i^{\,e_3} + \varepsilon_i \qquad [1]$$

with $\quad y_i = \dfrac{d_i}{D}$ and $\quad x_i = \left(\dfrac{H - h_i}{H - 1,3} \right)$

Where,
d_i = is the diameter under-bark (cm).
h_i (m) = height in the stem.

D = is the diameter over-bark at breast height (cm).
H = is the total height of the tree (m), b_i and e_i are the parameters of the model, and ε_i is the residual.

Basic wood density modeling. In the modeling of basic wood density two alternatives were considered, taking a three parameters polynomial model as a base, which best describes the relationship between the wood basic density and tree heights (Fang and Yang, 2003) (model 2). The first consisted of the average model fitting; the second consisted of a single tree model fitting, in which the coefficients a_i were estimated based on tree state variables, *i.e.* $a_i = f$ ($1\ D\ H\ D^2\ H^2\ DH\ D^2H\ HC\ D^2H_C$).

$$\rho_i = a_0 + a_1 x_i^{0,5} + a_2 x_i^{3} + \varepsilon_i \qquad [2]$$

Where,
ρ_i = is the basic density of the wood (kg m^{-3}) at the relative height x_i in the stem, a_i are the model parameters and ε_i is the residual.

Models fitting and comparison. In the fitting of the models the SAS statistical software (SAS 2009) was used. In the linear model fitting the PROC REG was used, while the fitting of the second alternative stem profile model was performed using PROC NLIN in SAS. This procedure fits nonlinear regression models, segmented models or robust regression, and estimates the parameters by nonlinear least squares or weighted nonlinear least squares. Furthermore, it gives reasonable starting values for successful convergence. The selection of predictor variables, in the fitting of individual tree stem profile and basic wood density models, was performed with the PROC REG and SELECTION = STEPWISE, a conditioning with the intention that each final model might include the predictor variables typical of the best average stem profile model, *i.e.* $x_i^{e_1}$, $x_i^{e_2}$ and $x_i^{e_3}$, and, respectively, the average basic wood density model, *i.e.* $x_i^{0,5}$ and x_i^{3}. Model comparison was made using the root mean square residual (3) and Akaike's index (4) (AIC; Akaike 1974, Wagenmakers and Farrell 2004). The model validation was performed based on the accuracy (5) (Bruce 1975).

$$RMSE = \sqrt{SSE/(n-p)} \qquad [3]$$

$$AIC = n\ln(SSE/n) + 2p \qquad [4]$$

$$E = \sqrt{\bar{\varepsilon}^2 + s^2} \qquad [5]$$

Where,
SSE = is the sum of squared residuals (*i.e.* $\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$).
p = is the number of model parameters.
n = is the total number of observations.
$\bar{\varepsilon}$ = is the average deviation (*i.e.* $\sum_{i=1}^{n}\varepsilon_i/n$).
s^2 = is the variance of residuals (*i.e.* $\sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon})^2/(n-1)$).

Stem biomass estimation. Stem biomass was determined from the double integral of the product between the stem cross-sectional area, obtained from a taper model, and the basic density of the wood (Parresol and Thomas 1989) (model 6).

$$\hat{w}_i = (1,3 - H)\int_{x_l}^{x_u}\int_{0}^{f(x_i)}\hat{\rho}_i(x_i, y_i)\partial y\partial x \qquad [6]$$

Where,
$\hat{w}_i$ = is the dry weight (kg) of the stem section between the relative heights x_l and x_u.
$f(x_i)$ = is the equation of the transversal area of the stem, obtained from the stem profile model, *i.e.*
$f(x_i) = kD^2[b_1 x_i^{e_1} + b_2 x_i^{e_2} + b_3 x_i^{e_3}]$ and $k = \pi/40000$
$\hat{\rho}_i$ = is the basic wood density equation (*i.e.* $\hat{\rho}_i = a_0 + a_1 x_i^{e_1} + a_2 x_i^{e_2}$). Thus, the general solution of the double integral was defined as follows (model 7):

$$\hat{w}_i = (1,3 - H)kD^2\left\{\begin{array}{l} b_1 x_i^{e_1}\left(\dfrac{(a_0 x_i}{e_1 + 1} + \dfrac{a_1 x_i^{1,5}}{e_1 + 1,5} + \dfrac{a_2 x_i^{4}}{e_1 + 4}\right) \\[2mm] + b_2 x_i^{e_2}\left(\dfrac{a_0 x_i}{e_2 + 1} + \dfrac{a_1 x_i^{1,5}}{e_2 + 1,5} + \dfrac{a_2 x_i^{4}}{e_2 + 4}\right) \\[2mm] + b_3 x_i^{e_3}\left(\dfrac{a_0 x_i}{e_3 + 1} + \dfrac{a_1 x_i^{1,5}}{e_3 + 1,5} + \dfrac{a_2 x_i^{4}}{e_3 + 4}\right) \end{array}\right\}\Bigg|_{x_u}^{x_l} \qquad [7]$$

The analysis of the biomass predictive model (model 7) was performed contrasting the real biomass estimates, which were obtained from the data of size and spacing between stem disks and basic wood density (8). Model 8 is the product of the solid volume of stem section, estimated by means of the formula of Smalian, *i.e.* $v = (A_b + A_s) \cdot L/2$, and the average basic wood density of the section, a value corresponding to the average of the densities of the stem disks at the ends of each section weighted by the corresponding sectional areas, *i.e.* $\bar{\rho} = (\rho_b A_b + \rho_s A_s)(A_b + A_s)$.

$$\hat{w}_r = \left[\frac{A_b \rho_b + A_s \rho_s}{2}\right] \cdot L \qquad [8]$$

Where,
w_r = is the dry weight (kg) of the stem section between two successive stem disks in the trunk of a tree, which was assumed as the actual value for the purposes of this study.
A_b, A_s and ρ_b, ρ_s, = respectively, are the section areas (m^2) and the basic densities of the wood (kg m^{-3}) of the basal and upper stem disks of each stem section.
L = is the length of the section (m).

The analyses of the predictive quality of the biomass model were made at tree level and log level. The biomass of each tree was obtained from the summation of the biomass of the sections along the stem. In both cases, the analyses included the separation according to size classes, *i.e.* small-end diameter classes for logs and DBH classes

for trees. Analyses were carried out in four different scenarios, resulting from combining two stem profile models, *i.e.* the best average stem profile model and the individual tree profile model, with the two basic wood density models, *i.e.* the average and the individual tree models. Thus, the first scenario resulted from the combination between the best average stem profile model and the average basic wood density model; the second scenario combined the best average stem profile model and the individual tree basic density wood model; the third scenario combined the individual tree stem model profile and the average basic wood density model; the fourth scenario resulted from combining the individual tree models for both stem profile and basic wood density. Analyses were made based on accuracy (*E*).

RESULTS

Stem profile modeling. The average fitting of the basic model of Bruce *et al*. (1968) was unsatisfactory. Although the estimators of the regression coefficients were significant (table 2), graphical analyses show that the model is not flexible enough to properly emulate the trend observed from the stem profile in this variety of poplar; the model tends to overestimate in the middle-upper portions of the stem and in that close to the base of the trees, and tends to underestimate the middle-lower part (figure 1). That deficiency, which is evident in the sample used for the fitting (figure 1, left), is also evident when using the model in each sampled stand (figure 1, right).

The average stem profile modeling significantly improved when estimating the exponents of the model from the available information. This result was achieved with a slight increase in the exponent e_1, *i.e.* from 1.5 to 1.6628, and significant increases in e_2 and e_3, *i.e.* from 3 to 5.3280

and, respectively, from 32 to 45.8188 (table 2). With these new exponent values, the model acquires sufficient flexibility to properly emulate the trend observed in the stem profile of this variety of poplar (figure 2; compared with figure 1), eliminating the shortcomings shown in the basic model of Bruce *et al*. (1968). Although this variant of the basic model of Bruce *et al*. (1968), as in the remainder of this document will be named "modified Bruce ", requires the estimation of six parameters, *i.e.* three coefficients and three exponents, it presented an index of Akaike 3.26 % smaller, the root mean square residual error was reduced by 9.45%, whereas the accuracy increased by 9.44 %. The best fitting of the model, which is evident in the sample used for the fitting (figure 2, left), is also evident on using the model in each particular stand (figure 2, right).

Incorporating tree state variables significantly improved stem profile modeling. The fitting at individual tree level, using the set of exponents determined as suitable for the average stem profile model for this variety of poplar, reduced in 5.74 % the mean square root residual error compared to the average model and exponents estimated from the available data, and in 14.66 % in comparison with the average model of Bruce *et al*. (1968). The reduction in the Akaike's index reached, respectively, 1.82 % and 5.14 %, and the accuracy increased 96.56 % and 96.20 %. In all models, the larger residuals are observed at the base of the stems due to the deformations that they present in that area (figure 3). The variables of the state of the tree with significant effect on coefficients b_1 and b_3 of the stem profile model were *H, DH, D²H,* y *H²*; no effect was detected on tree size upon the coefficient b_2 (table 2).

Basic wood density modeling. Although average model fitting generated significant parameters (table 3), seeming to

Table 2. Estimators and statistics from the fitting of the stem profile models.
Estimadores y estadísticos del ajuste de los modelos de perfil fustal.

Parameters	Average model		Individual tree model
	Bruce *et al*. (1968)	Modified Bruce	
e_1	1.5	1.6628	1.6628
e_2	3	5.3280	5.3280
e_3	32	45.819	45.819
b_1	1.0571 [*]	1.0890 [*]	$0,82535^{[**]} + 0,000655022DH^{[**]} - 0,00001245D^2H^{[**]}$
b_2	-0.2120 [*]	-0.2554 [*]	$-0.26359^{[**]}$
b_3	0.08354 [*]	0.0645 [*]	$0,083253^{[**]} - 0,00329291H^{[**]} + 0,000087504DH^{[**]}$
Statistics of fitting			
RMSE	0.0475	0.0429	0.04052
AIC	-6232.2	-6435.41	-6552.47
E (Fitting)	0.04743	0.04295	0.001632
E(Validation)	0.03171	0.02678	0.02764

*: *P* < 0.05; **: *P* < 0.001; *RMSE*: root of the mean square residual; *AIC*: Akaike index; *E*: accuracy.
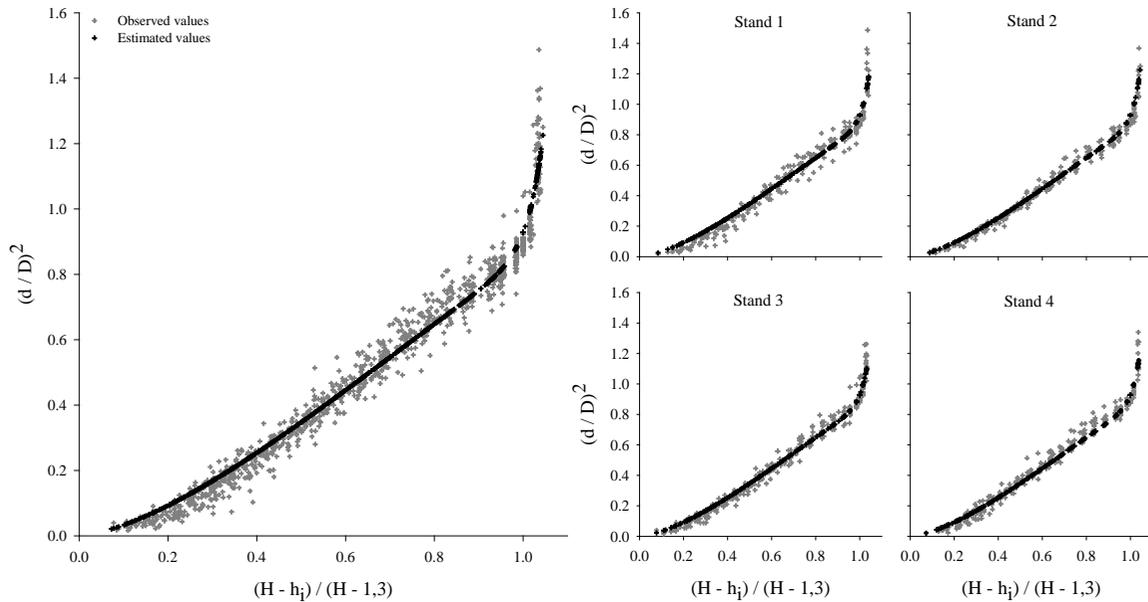
Figure 1. Graph of the basic stem profile model of Bruce *et al.* (1968) resulting from the average fitting, showing in gray the observation and in black the estimation curve. Left: the estimation curve and the observations of the four stands used in the model fitting; right: the estimation curve and the observations of each stand in the sample.

Gráfica del modelo de perfil fustal básico de Bruce *et al.* (1968) resultante del ajuste promedio, mostrando en gris las observaciones y en negro la curva de estimación. Izquierda: la curva de estimación y las observaciones de los cuatro rodales utilizadas en el ajuste del modelo; derecha: la curva de estimación y las observaciones de cada rodal en la muestra.
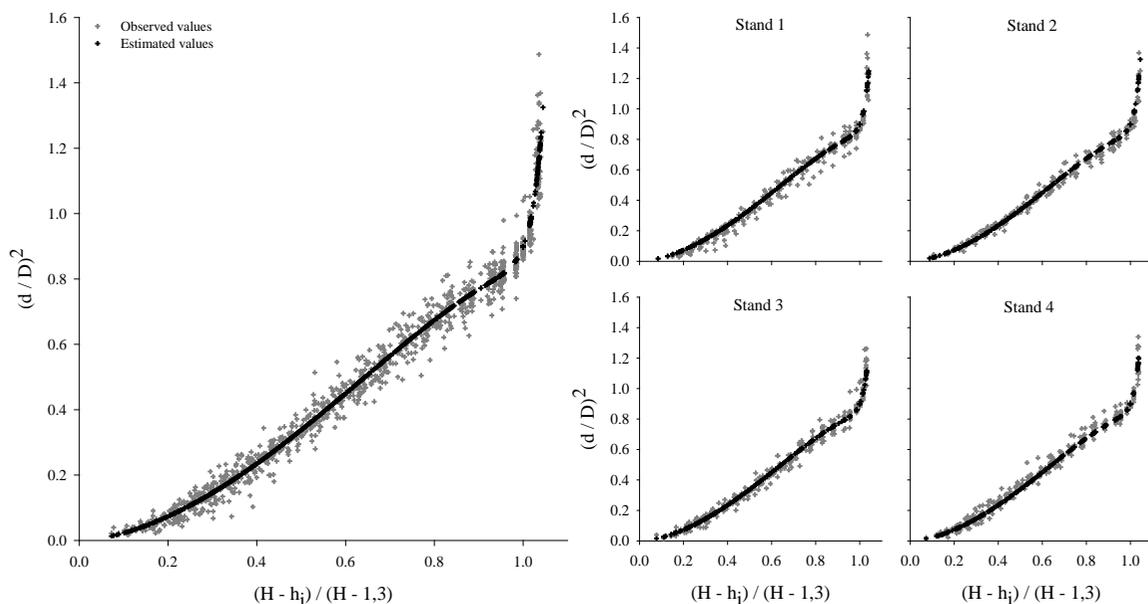
Figure 2. Graph of basic stem profile model Bruce *et al.* (1968) resulting from the fitting average, but estimating the model exponents e_i of the model from the stem taper data available for this study, *i.e.* "Bruce modified" model. The observations are shown in gray; in black, the estimation curve. Left: the estimation curve and the observations from the four stands used in the model fitting; right: the estimation curve and the observations from each stand in the sample.

Gráfica del modelo de perfil fustal básico de Bruce *et al.* (1968) resultante del ajuste promedio, pero estimando los exponentes e_i del modelo a partir de la información de ahusamiento fustal disponible para este estudio, *i.e.* modelo "Bruce modificado". Las observaciones se muestran en gris; en negro, la curva de estimación. Izquierda: la curva de estimación y las observaciones de los cuatro rodales utilizadas en el ajuste del modelo; derecha: la curva de estimación y las observaciones de cada rodal en la muestra.
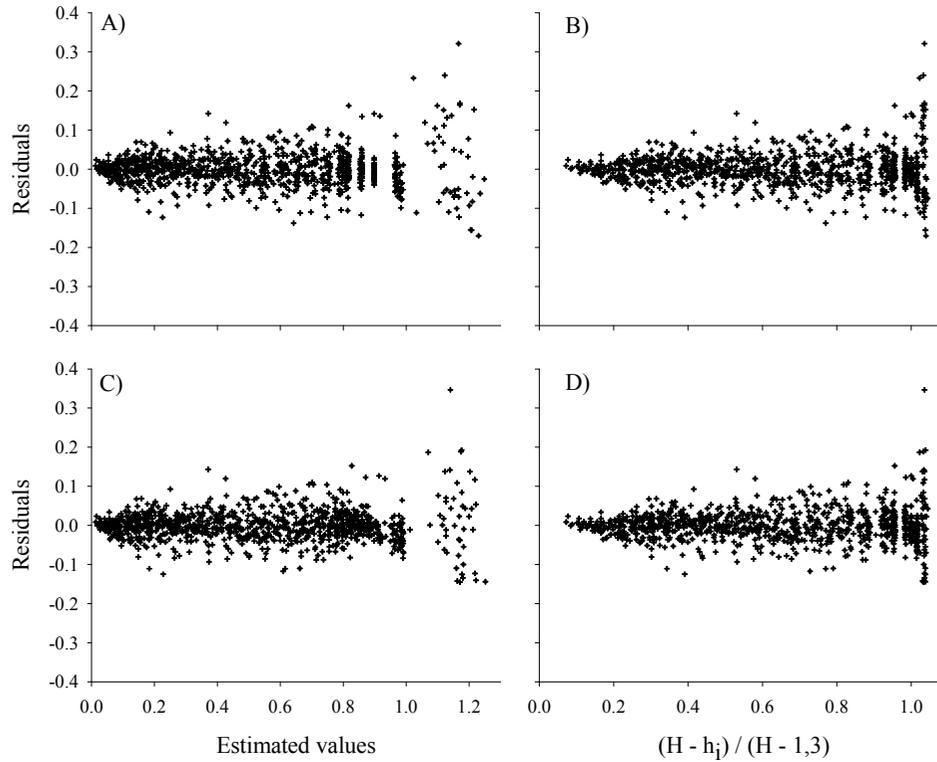
Figure 3. Graphic of stem profile residuals. Left: residuals against estimated values; right: residuals against stem relative height. A) and B) fitting of modified Bruce model; C) and D) fitting of the individual tree model.

        Gráfica de residuos. Izquierda: residuos contra valores estimados; derecha: residuos contra altura fustal relativa. A) y B): ajuste del modelo de Bruce modificado; C) y D) ajuste del modelo de árbol individual.

Table 3. Estimators and statistics of basic wood density models.

        Estimadores y estadísticos del ajuste de los modelos de densidad básica de la madera.

Parameters	Average model	Individual tree model
a_0	453.7000[*]	454.3532[**]
a_1	-89.06250[*]	$-143{,}372^{[**]} + 9{,}21683D^{[**]} - 0{,}22431DH^{[**]}$
a_2	-18.65460[*]	-18.72341[**]
Fitting statistics		
RMSE	15.65760	12.25616
AIC	2549.382	2324.565
E (Fitting)	15.62377	12.20312
E (Validation)	13.67323	12.31957

*: $P<0.05$; **: $P<0.001$; *RMSE*: root of mean square error; *AIC*: Akaike index; *E*: accuracy.

adequately represent the sinusoidal trend of increasing basic density of wood from the base to the apex of the variety of poplar studied (figure 4, left), the model is not satisfactory to be used at stand level (figure 4, right). Thus, the presence of a local effect important in the determination of the basic density of the wood is evident, which can be explained, at least partially, incorporating variables of the state of the tree.

The incorporation of tree state variables significantly improves the modeling of the basic wood density. The fitting at individual tree level reduced by 21.72 % the average squared residual root in comparison with the average model. The reduction in the Akaike's index reached 8.82 %; the accuracy increased 21.89 %. In both models, the greater residues are observed at the base of the stems (figure 5). The state
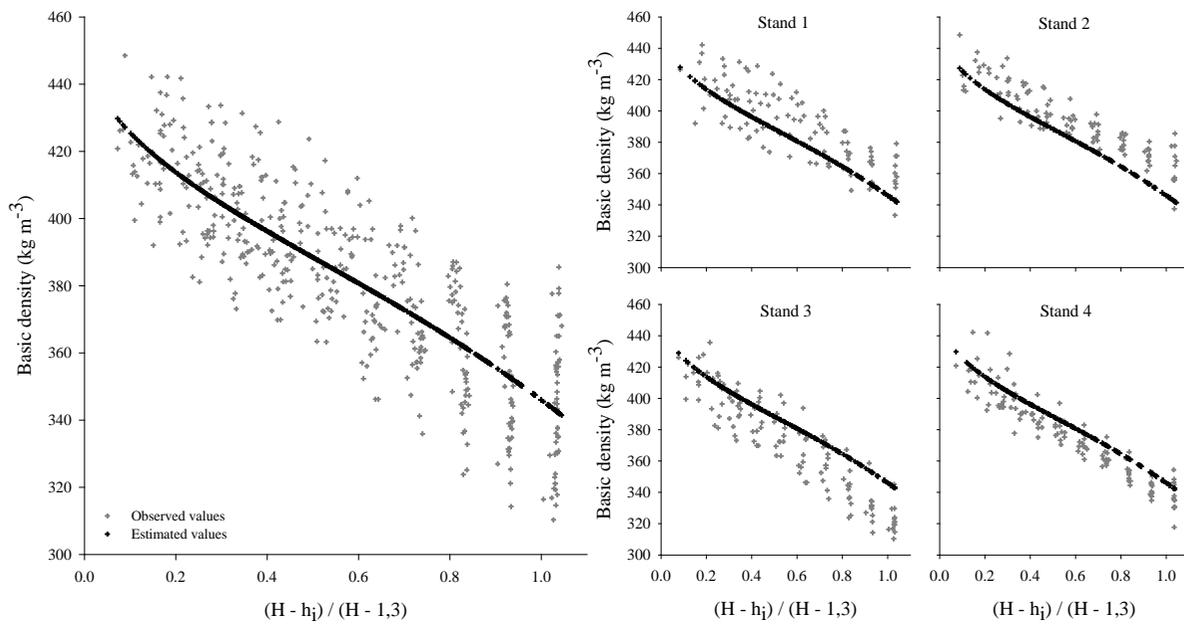
Figure 4. Graph of the basic wood density model resulting from average fitting. Observations are shown in gray; in black, the estimation curve. Left: the estimation curve and observations from the four stands used in model fitting; right: the estimation curve and observations from each stand in the sample.

Gráfica del modelo de densidad básica de la madera resultante del ajuste promedio. Las observaciones se muestran en gris; en negro, la curva de estimación. Izquierda: la curva de estimación y las observaciones de los cuatro rodales utilizadas en el ajuste del modelo; derecha: la curva de estimación y las observaciones de cada rodal en la muestra.
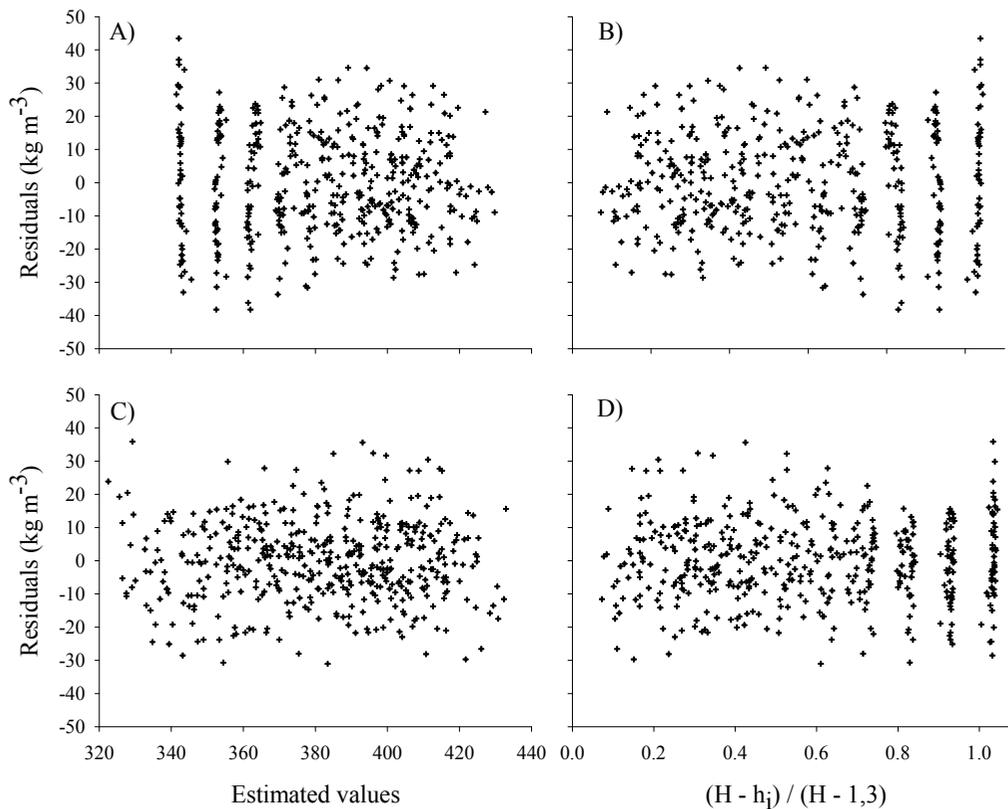
Figure 5. Graphic of basic wood density residuals. Left: residuals against estimated values; right: residuals against relative stem height. A) and B) fitting of the average basic wood density model; C) and D) fitting of the individual tree model.

Gráfica de residuos. Izquierda: residuos contra los valores estimados; derecha: residuos contra altura fustal relativa. A) y B): ajuste del modelo promedio de densidad básica de la madera; C) y D) ajuste del modelo del árbol individual.

variables of the tree with significant effect on the coefficient a_1 of basic wood density were D and DH; no tree size effect was detected on the coefficients a_0 and a_2 (table 3).

Stem biomass estimation. The predictive quality of the biomass equation varied depending on the scenario of analysis, *i.e.* on which stem profile (model 1) and basic wood density (model 2) equations are combined in the double integral (model 7). Both at log level and at tree level the superior accuracy was achieved with the combination of individual tree equations (scenario 4). In the analyses at log level (table 5), carried out on the sample used in the fitting of stem profile and basic wood density, the combination of individual tree models achieved an average accuracy of 3.5985 kg log^{-1}, *i.e.* 4.2 % more accurate than scenario 2; even less accurate, and in descending order, were the combinations of scenarios one and three. No evidence of significant effect of the size of logs on the accuracy was detected. These results agree with those obtained in the validation sample, except for the slight advantage of scenario 3 over 2 and a slight tendency to decrease the accuracy in large diameter logs. It is likely that the slight advantage of scenario three over two is due to the unrestricted random choice of the validation sample, which included individuals of smaller DBH and greater variability than those present in the fitting sample (compare both samples from stand 2, table 1).

In the analyses at tree level (table 4), carried out on the sample used in the fitting of stem profile and basic wood density equations, the combination of individual tree models (scenario 4), achieved an average accuracy of 23.6 kg tree^{-1}, *i.e.* 6.50 %, 8.03 % and 8.83 % more accurate than the combinations of models considered in scenarios two, one and three respectively. There was no evidence of significant effect of tree size on the accuracy of the estimation of stem biomass. These results coincide with those obtained in the validation sample, except the slight advantage of scenario three over two, which would be explained by the smaller DBH size of a stand in the validation sample, as mentioned in preceding paragraphs.

DISCUSSION

Stem diameter varies not only along the stem but also among species and among individuals of the same species. Thus, a major problem in estimating stem diameter at specific stem points is to fit a taper model that is applicable to trees of different sizes (Earikäinen 2001). The results of this study allow the assertion that, in the variety of poplar analyzed, the bigger difficulty in modeling stem taper is located in the lower part of the stem, by the larger irregularities which it presents in this area. The exponents of the average basic model of Bruce *et al.* (1968) do not give enough flexibility to follow the trend observed for the stem profile and, as a result, the average resulting equation considering these exponents generates overestimations and underestimations in extensive areas of the stem; results that agree with those observed by Barrio *et al.* (2007), who found the same pattern of deficiency in diameter along the stem when they used an average stem profile model to calculate the diameter and volume of *Populus x eurameri-cana* trees. A significant increase in accuracy is achieved by estimating the value of exponents of the stem profile model from the data available; the average equation fitted considering these new exponents properly emulates the taper trend data. The incorporation of state variables of the tree also produces accuracy increase of the fitted equation. Thus, the most precise profile model results from simultaneous individual fitting, with exponents estimated from specific taper data from the species in question.

Basic wood density, defined as the dry wood mass per unit volume of green wood, is a characteristic of high variability. Among others, it varies with age, environmental conditions, availability of water and nutrients in the soil, silvicultural management and among species (Prado and Barros 1989, Kohyama and Hotta 1990, Harris and Cown 1991, Downes *et al.* 1997, Zobel and Sprague 1998, Peredo *et al.* 2007, Moya *et al.* 2009). In turn, it can vary widely within the same tree, longitudinally from the pith to the bark (at different ages) and axially from the base to the apex (Zobel and Talbert 1988). According to the results of this study, the basic wood density of the *Populus* I - 488 variety increases from the base to the apex of the trees; results that agree with those reported by several authors in different types of hardwoods, including species such as *Populus deltoides* CV, *Acacia melanoxylon* R.Br., *Eucalyptus grandis* Hill ex Maiden, *Eucalyptus regnans* F. Muell., *Eucalyptus globulus* Labill and *Eucalyptus nitens* (Deane *et* Maiden) (Igartúa *et al.* 2002, McKinley *et al.* 2002, Fang and Yang 2003, Díaz *et al.* 2010, Igartúa and Monteoliva 2010). Other studies have reported for the genus *Eucalyptus* an initial reduction in basic density between the base and DBH height, and then an increase toward the apex (Bhat *et al.* 1990, Clark 2001, Raymond and Muneri 2001, Monteoliva *et al.* 2002). This trend was also observed in clones of the genus *Populus* (Beaudoin *et al.* 1992, Gutiérrez and Baonza 2001). The poplar variety considered in the present study did not present such behavior in the basal zone, but the pattern of variation for heights above DBH coincides with that reported in the cited studies. As a consequence, the modeling of the basic wood density of this variety of poplar could be carried out with models more parsimonious than in other varieties. In the modeling of the basic wood density along the stem, the individual tree model achieves higher accuracy than that achieved in the average model.

The estimation of the stem biomass combining stem profile models with basic wood density models is an attractive option not only for the high level of accuracy achieved but also for the flexibility of the resulting equation of the double integral. Although in the fitting of both models the same data as that required for fitting biomass models with fixed top merchantable diameter is used, the resulting equation is more flexible and useful than those

Table 4. Accuracy of estimation of total stem biomass by DBH class and scenario of analysis, both in the sample used for fitting of stem profile and basic wood density models as in the validation sample.

Exactitud de estimación de la biomasa fustal total, según clase diamétrica de árbol y escenario de análisis, tanto en la muestra de ajuste de los modelos de perfil fustal y densidad básica como en la muestra de validación.

	Sample used in models fitting					Validation sample					Full sample				
DBH class (cm)	Number of trees	Accuracy (kg tree^{-1})				Number of trees	Accuracy (kg tree^{-1})				Number of trees	Accuracy (kg tree^{-1})			
		Scenario 1	Scenario 2	Scenario 3	Scenario 4		Scenario 1	Scenario 2	Scenario 3	Scenario 4		Scenario 1	Scenario 2	Scenario 3	Scenario 4
20-25	4	10.578	6.9650	12.772	7.2719	3	8.2663	6.7168	12.039	9.454	7	8.9963	6.4855	11.088	6.9866
30	20	19.590	20.569	20.805	18.235	5	17.271	20.457	17.388	15.456	25	18.683	19.752	19.264	17.570
35	22	30.742	27.694	31.843	27.878	3	20.502	15.001	22.828	16.114	25	29.943	26.362	31.076	26.693
40	2	46.681	63.175	36.216	55.058	1	-	-	-	-	3	43.820	51.461	30.246	40.988
Average	48	25.763	25.269	26.006	23.637	12	19.674	20.004	16.899	13.638	60	24.689	23.710	24.471	22.029

Table 5. Accuracy of logs biomass estimation by log small-end diameter class and scenario of analysis, both in the sample used for fitting of stem profile and basic wood density models as in the validation sample.

Exactitud de estimación de la biomasa por troza, según clase de diámetro menor de troza y escenario de análisis, tanto en la muestra de ajuste de los modelos de perfil fustal y densidad básica como en la muestra de validación.

	Sample used in models fitting					Validation sample					Full sample				
Small-end diameter class (cm)	Number of logs	Accuracy (kg log^{-1})				Number of logs	Accuracy (kg log^{-1})				Number of logs	Accuracy (kg log^{-1})			
		Scenario 1	Scenario 2	Scenario 3	Scenario 4		Scenario 1	Scenario 2	Scenario 3	Scenario 4		Scenario 1	Scenario 2	Scenario 3	Scenario 4
5	32	2.5038	2.5178	2.4626	2.4703	8	1.6776	1.6720	1.5701	1.5596	40	2.3760	2.3757	2.3486	2.3443
10	63	1.8972	1.9210	1.8427	1.8476	19	1.5303	1.4816	1.4353	1.3696	82	1.8239	1.8223	1.7816	1.7680
15	84	2.7068	2.7840	2.6804	2.6963	23	2.0361	1.9189	2.0351	1.8362	107	2.5638	2.6016	2.5400	2.5334
20	84	3.7248	3.6913	3.8337	3.5690	20	3.3066	3.2080	3.1199	2.7421	104	3.6523	3.5654	3.6954	3.4348
25	85	5.0239	4.8292	5.2209	4.5957	21	3.3429	3.4668	3.6328	3.1152	106	4.7338	4.5171	4.8933	4.3329
30	62	5.3431	5.1783	5.5327	4.9571	12	4.2786	4.4901	4.0231	3.4589	74	5.1865	4.9469	5.2953	4.7070
35	6	5.4714	3.6332	4.2122	2.6876	2	5.1686	5.9913	1.7192	0.9999	8	5.3928	4.0024	3.7926	2.3391
Average		3.8637	3.7583	3.9465	3.5985		2.8459	2.8940	2.7392	2.3944		3.6928	3.5516	3.7284	3.3946

of fixed top merchantable diameter and this, together with the stem profile model, constitutes a simple set of equations that allows complementary volumetric-mass quantification of the stem. For example, the taper equation can be used for the volumetric quantification of the fraction of stem with suitable dimensions to generate logs destined for sawmilling and laminated and rotated veneer; the biomass equation can be used for the mass quantification of the remaining stem that could be used for bioenergy. Obviously, the accuracy of the biomass estimates depends on the precision of both the stem profile and basic wood density models; according to the results obtained in the present study, in both cases it is advantageous to incorporate tree state variables, *i.e.* by developing individual tree models. This is reflected in the best estimate of stem profile and basic wood density, and solves the problems of over- and underestimation observed in other methods (details at Barrio *et al.* 2007).

Diameter at breast height (DBH) and the total tree height were adequate state variables to explain residual variability induced by individual-tree effects on both stem taper and basic wood density. These pairs of easy-to-measure tree variables are the same used by traditional indirect methods for estimating stem biomass (see *e.g.* Brown *et al*. 1989, Myers 1990, Husch 2001, Diéguez *et al.* 2003, Specht and West 2003, Pérez and Kanninen 2002, Fonseca *et al*. 2009) and both are captured by customary forest inventory; thus, no additional effort is required to practical use of the flexible biomass equation presented in this paper.

CONCLUSIONS

Most accurate stem diameter and basic wood density estimates are obtained from individual tree models. Thus, the most accurate estimates of biomass, based on combinations of stem profile and basic wood density models, are obtained combining individual tree models. The flexible equation of stem biomass obtained for the *Populus x canadensis*, I-488 variety not only estimates stem biomass with high precision, but also enables the estimation of the biomass between any two points of the stem, which represents a great advantage compared with traditional biomass models which only estimate the total biomass of the stem or up to a specific top merchantable diameter. In addition, this equation, together with the stem profile model, constitutes a simple system of equations that allow complementary mass-volume quantification of the stem. Thus, depending on the final use of timber and on the dimensions of the tree, a fraction of the stem can be quantified in units of volume and the remaining fraction in weight units.

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